MEEN 364 ***Spring 2018***

Assignment: Lab 5, 8, and 9 Report	
Instructor: Dr. Prabhakar Pagilla	
Section: 501	
Submission Date: 23 April 2018	

All authors have contributed to the preparation of the report and have read the final version of the report.

On my honor, as an Aggie, I have neither given nor received unauthorized aid on this academic work.

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I. Abstract

These three labs, Labs 5, 8, and 9, revolved around a real world problem of creating and implementing a system to control the level and flow rate of a coupled tank. This was done by first finding the mathematical model and numerical parameters of the coupled tank system. Then a simulation was ran so that a controller could be determined and refined through rapid iterations. From the information gathered in the first section of this lab, the PID controller for the system was initially determined to have the theoretical gain values of K_p = 6.7135, K_i = 1.0078, and K_d = 67.2116. After some refinement to make the model more linear, these values were adjusted to K_p = 35.7135, K_i = 0.0700, and K_d = 107.2116. Finally, this controller was implemented to manage both the level and the rise of water in the tank. In order for this controller to meet the specifications of a rise time being less than 15 seconds, a maximum overshoot being less than 20%, and a steady state error of 0, these gains were once again adjusted to the experimental values of K_p = 35.7135, K_i = 0.1000, and K_d = 107.2116. As a result of all of these adjustments, a controller was created that could adequately control the coupled tank system.

II. Introduction

In this series of experiments, a two-tank system was modeled with the goal of designing and implementing a controller to maintain the liquid level of the coupled tank system. The goals of the first lab, Lab 5, were to derive the nonlinear mathematical model of the tank system, determine the numerical parameters of the system, then calibrate the pressure transducers for further testing. Next, Lab 8 was conducted with the goals of designing a closed loop controller for the linearized model of the system using MATLAB and Simulink, tweaking the gains to meet the nonlinear system model, and understand the control design process for real-world systems. The final goal in Lab 9 was to use these models and data to implement the PID (Proportional, Integral, Derivative) controller to meet the desired specifications of Rise Time < 15s, Max Overshoot < 20%, and 0 steady state error. The conclusions drawn from these experiments have many real life applications including refining processes that use reservoirs and flow, modelling nonlinear systems such as AC power flow, and designing a controller for any of these nonlinear systems.

III. Theory

The purpose of labs 5, 8, and 9 was to develop a control system to control the level of water in a pair of coupled tanks. The tank system consists of a pump to adjust the flow rate to the top tank (q_i). Water is pumped into the upper tank (Tank 1) which is connected to the lower tank (Tank 2) by a sink at point b through which there is a flow rate of q_{12}. The water then flows out the bottom of Tank 2 into a reservoir which is connected to the pump. This entire system can be seen in *Figure 1* on the next page.

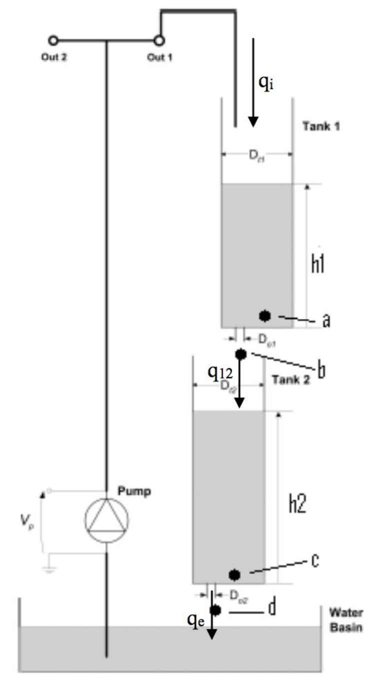

Figure 1: Quanser Coupled Tank System[1]

Pressure transducers at the bottom of each tank give voltage signals proportional to the height of water in the tanks. The pump produces a flow rate based on the voltage output to the pump. The linearized model of the system developed in Lab 5 is shown in equations 1 and 2 below. Equation 3 defines the flow rate, q_i, produced by the pump in terms of the motor constant k and the voltage output to the pump, v_i.

$$A_1 \frac{d\delta h_1}{dt} + \frac{c_1 \delta h_1}{2\sqrt{h_{10}}} = \delta q_i \quad (1)^{[2]}$$

$$\frac{c_1 \delta h_1}{2\sqrt{h_{10}}} - \frac{c_2 \delta h_2}{2\sqrt{h_{20}}} = A_2 \frac{d\delta h_2}{dt} \quad (2)^{[2]}$$

$$\delta q_i = k \delta v_i \quad (3)^{[2]}$$

$$\frac{c_1}{2\sqrt{h_{10}}} = G_{v_1} \ ; \ \frac{c_2}{2\sqrt{h_{20}}} = G_{v_2} \quad (4)^{[2]}$$

In equations 1 and 2, A stands for the cross sectional area of each tank, h stands for the height of the water in each tank, v_i is the voltage output to the pump, and other terms are constants of the coupled tanks system. Equation 4 defines the terms G_{v1} and G_{v2} shown in equations 1 and 2.

In Lab 8, a PID controller must be created to control the water level in Tank 2 to have a rise time of less than 15 seconds, a less than 20% overshoot, and a steady state error of 0. This PID controller is shown in *Figure 2* below.

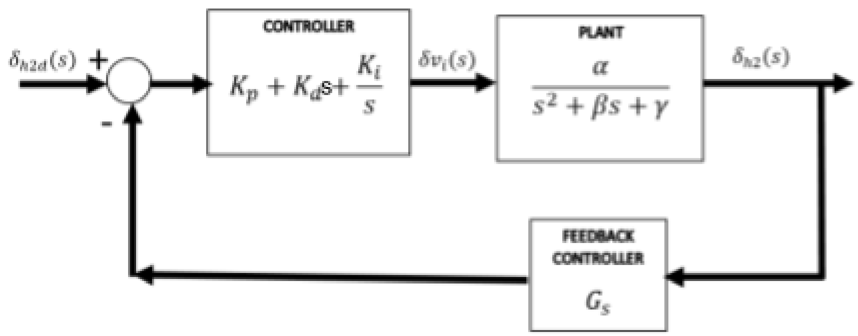

Figure 2: PID Controller for the Coupled Tank System[2]

The relationship between the voltage output to the motor and the height of Tank 2 is shown in equation 5 below where P(s) represents the plant function.The plant function, derived in Lab 8, governing this relationship is shown fully in equation 6 below.

$$\delta h_2(s) = P(s)\delta v_i(s) \qquad (5)^{[2]}$$

$$\delta h_2(s) = \frac{\frac{G_{v_1} k}{A^2}\delta v_i(s)}{s^2 + \left(\frac{G_{v_1} + G_{v_2}}{A}\right)s + \frac{G_{v_1} G_{v_2}}{A^2}} \qquad (6)^{[2]}$$

With the linearized system shown in equation 6, the PID for controlling the system can be designed. This is done through dominant pole approximation, which allows for the use of the governing equations defining rise time, percent overshoot, and settling time. Pole placement can then be used to determine the gains K_p, K_d, and K_i. This gives the ability to design around rise time, percent overshoot, and steady state error specifications.

In the final portion of this lab, the controller is implemented in the coupled tanks system. The original control gains are used initially, and then, depending on the performance of the real system, can be changed to optimize performance of the system.

IV. Procedure

In the first part of the experiment, we used Simulink to determine the flow rate correction values c_1 and c_2 of the pump by recording heights in the tanks at 5 voltages ranging from minimum flow voltage to maximum voltage without overflowing, allowing the system to reach steady state between each trial voltage. For each voltage, we plotted the flow rate vs voltage to determine the motor gain constant k_m, and the flow rate vs square root of height data for each tank was plotted to determine c_1 and c_2. Lastly, we determined the linear calibration equation of the pressure transducer for tank two and plotted h_2 vs v_i to estimate steady state gain.

Once we completed the steps in Lab 5 outlined above, we moved on to designing and simulating the controller in Lab 8. First, we assumed a PID control structure and designed a controller (outlined below in *Figure 15*) to meet the linearized model specifications, which we then verified the gains by simulating in MATLAB. We then built Simulink models for both the linear and nonlinear models of the system described in the Theory section. For the linearized model, we simulated a step input with initial height in tank two of 10 cm with desired height of 12 cm. From there, we tweaked the gains to meet the specifications of Rise Time < 15s, Max Overshoot < 20%, and 0 steady state error.

Once we completed the design and simulation of Lab 8, we proceeded to implement our controller on the coupled tank system in Lab 9. We built a Simulink model to implement the controller we designed, as shown in *Figure 18*. We used the calibration data for our output voltage to flow rate and pressure transducer voltage to tank height reading to communicate with the pump and sensor on the rig. By varying $K_p,\ K_i,\ and\ K_d$, we were able to achieve the desired specifications around the desired operating point of 10cm in tank two.

V. Results and Discussion

Using the procedure outlined in the previous section, the pressure transducer was calibrated. The results of the calibration are shown in *Figure 3*, with the resulting linear regression giving the following relationship *Height = 6.2461*Transducer Voltage + 0.2566*.

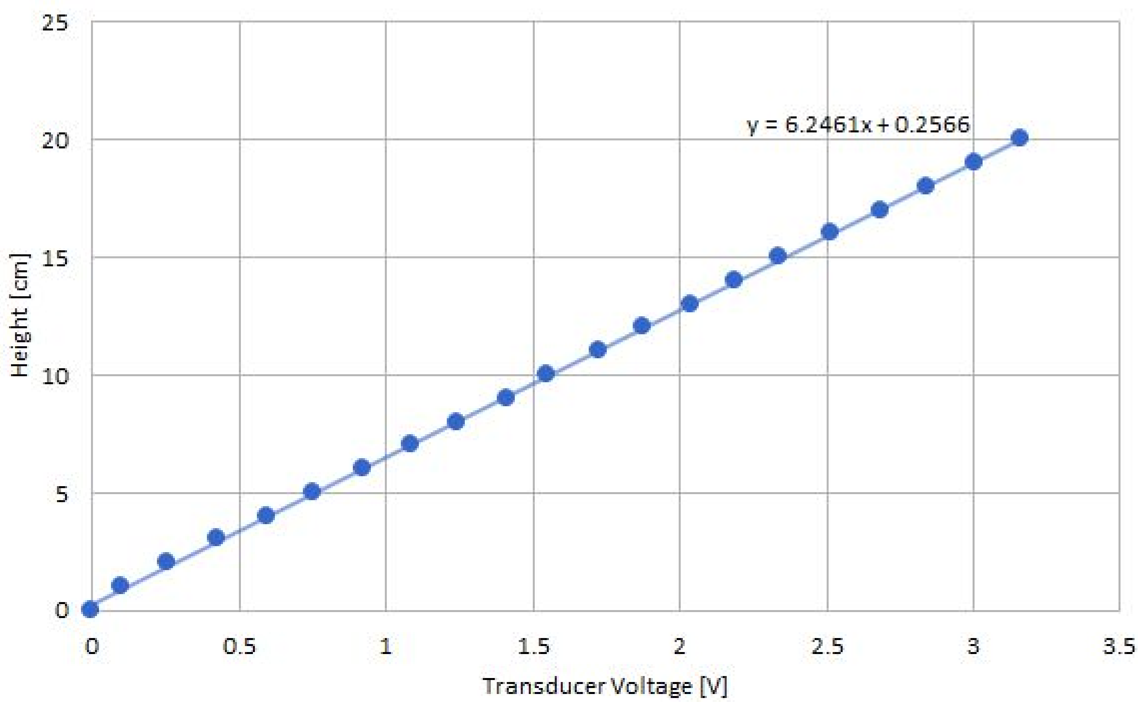

Figure 3. Transducer Calibration Plot

The pump produced different volumetric flow rates when given different voltages. The collected data on steady state heights and volumetric flow rates is summarized in *Table 3* of Appendix D. The relationships drawn from this data produced *Figures 4 - 7*, which gave the values shown in *Table 1*.

Table 1. Resultant System Parameters

Parameter	Value
C_1	6.5865
C_2	6.8562
K_m	1.9756
K_{pt}	2.2453

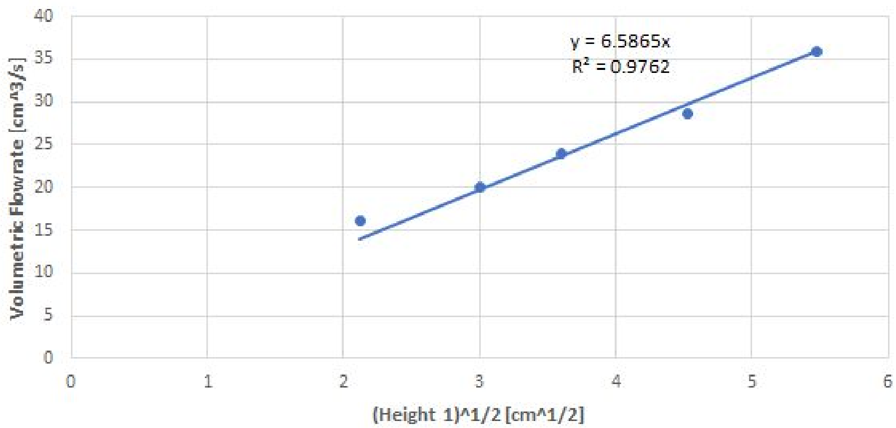

Figure 4. Q_i vs. $H_1^{1/2}$ Used in Determining Constant c_1

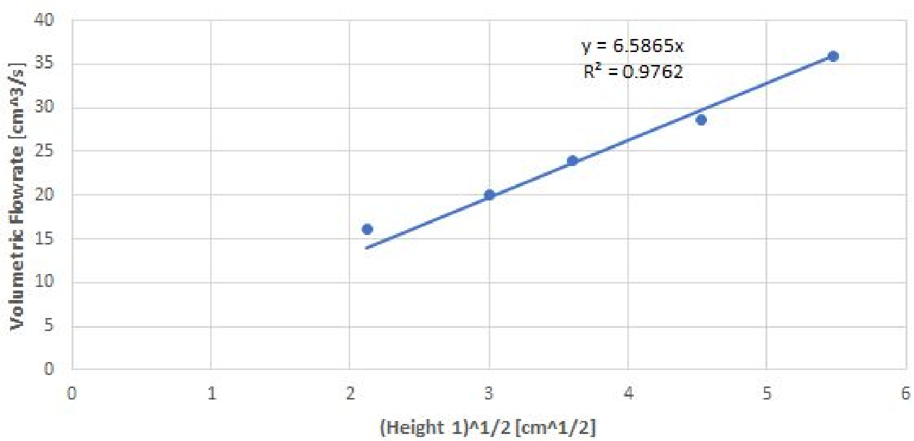

Figure 5. Q_i vs. $H_2^{1/2}$ Used in Determining Constant c_2

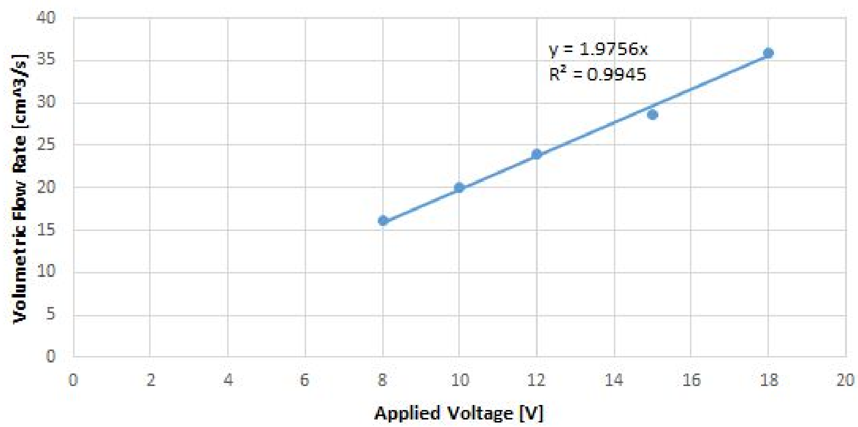

Figure 6. Q_i vs. V_i Used in Determining the Motor Gain Constant K_m

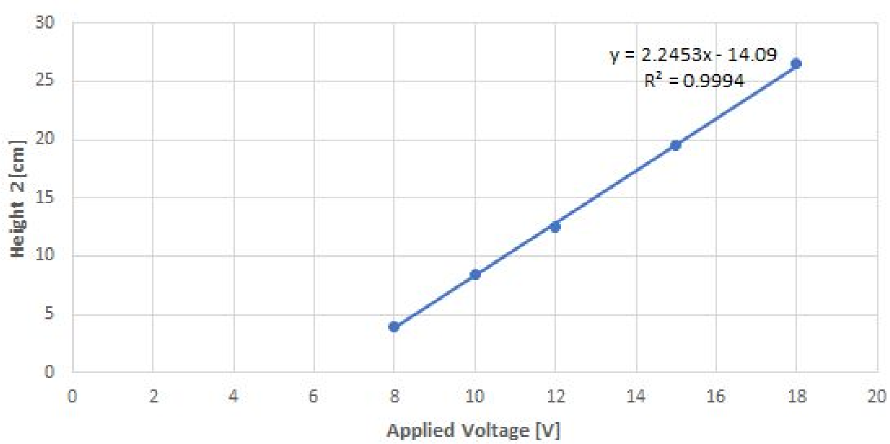

Figure 7. H_{2i} vs. V_i Used in Determining the Steady State Gain Constant K_{pt}

Using the parameters found in Lab 5, the script *estimate_param.m* estimated the control parameters for the PID controller gains to obtain the required performance of a rise time less than 15 seconds, less than 20% overshoot, and no steady state error. These parameters were tweaked through the linear model shown in *Figure 15* and the nonlinear model shown in *Figures 16 and 17*. The resultant parameters that best suited the design requirements are summarized in *Table 2*. It was found that the parameters that fit the linear model would be very close to fitting the nonlinear model. However, the steady state error and rise time were too large, and so adjustments were made in the value of K_i as shown.

Table 2. Control parameters found by various methods and models

	Original Estimate	Linear Model Optimization	NonLinear Model Optimization
K_p	6.7135	35.7135	35.7135
K_i	1.0078	0.0700	0.1000
K_d	67.2116	107.2116	107.2116

Using each set of gains found in the simulated models, experiments of the coupled tank system were run. The results of the experiments are shown in *Figures 8 - 13* and are compared to the simulations using the same parameters.

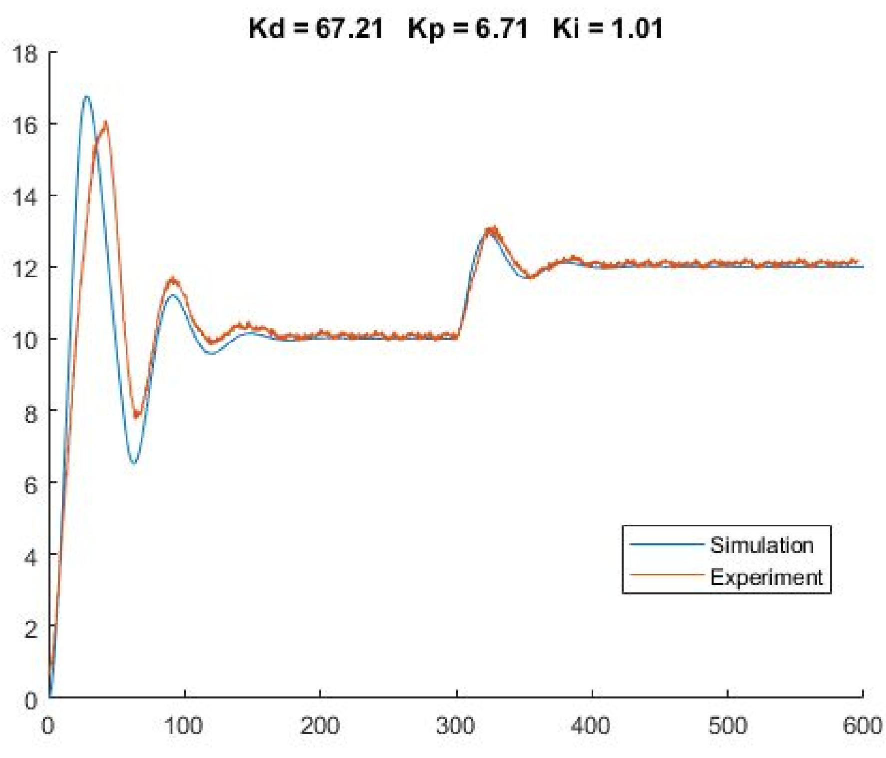

Figure 8. Linear Response with Estimated Parameters

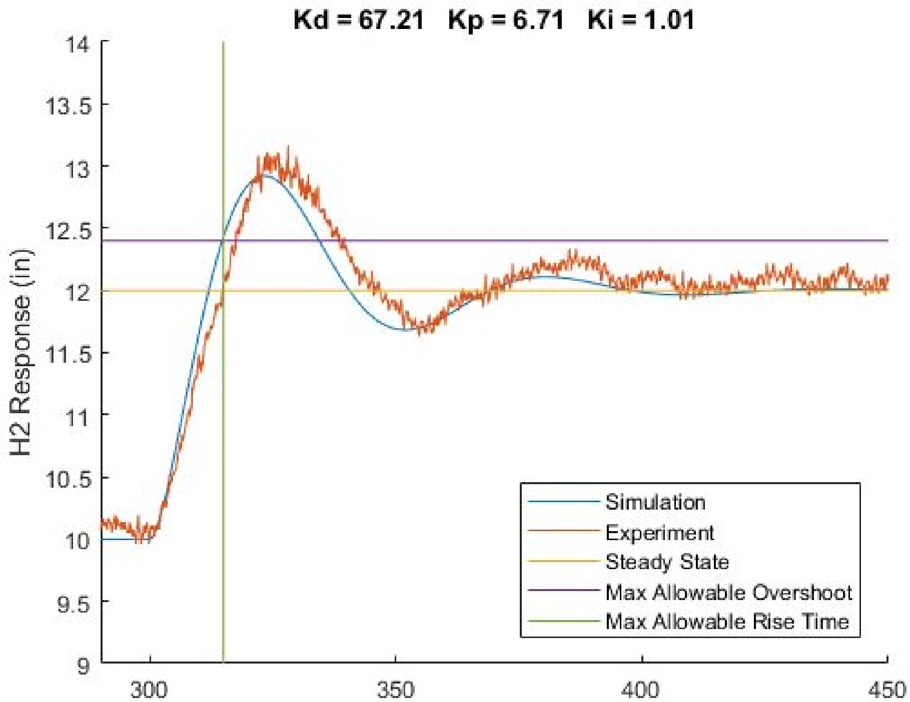

Figure 9. Linear Response with Estimated Parameters in Area of Interest

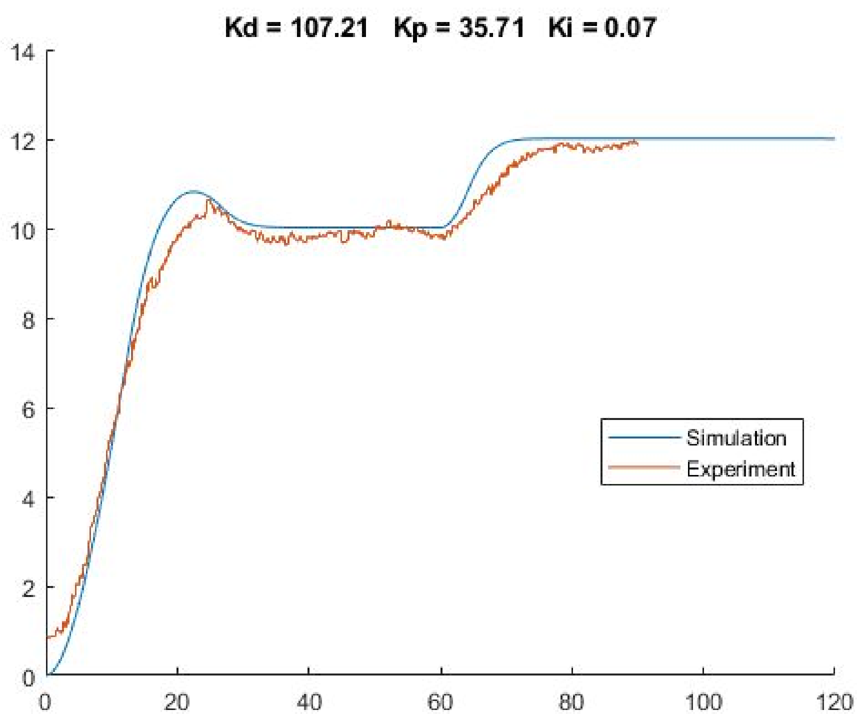

Figure 10. Linear Response with Optimized Parameters

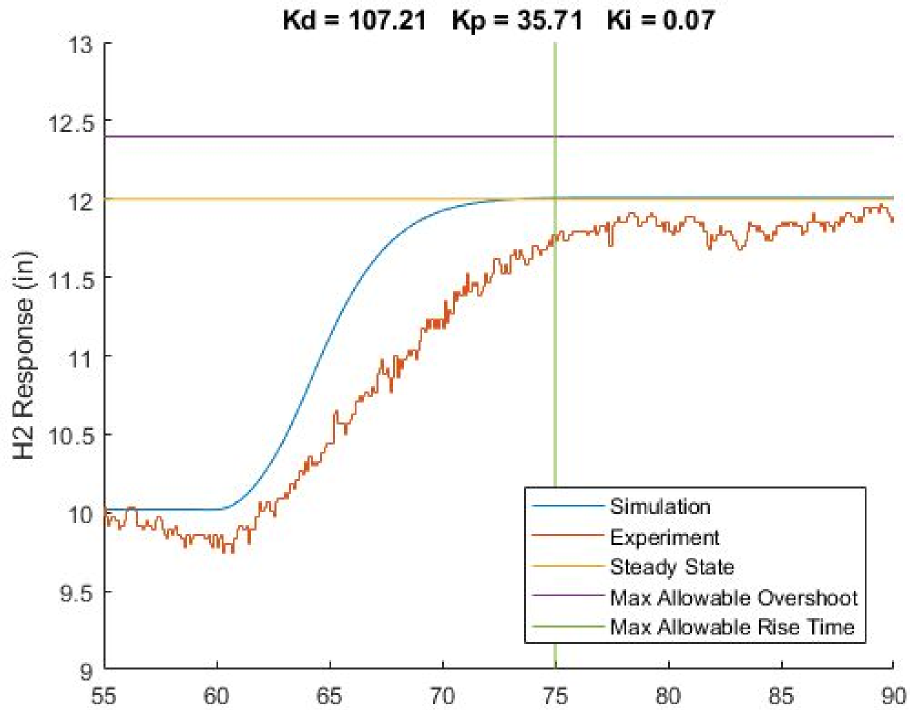

Figure 11. Linear Response with Optimized Parameters in Area of Interest

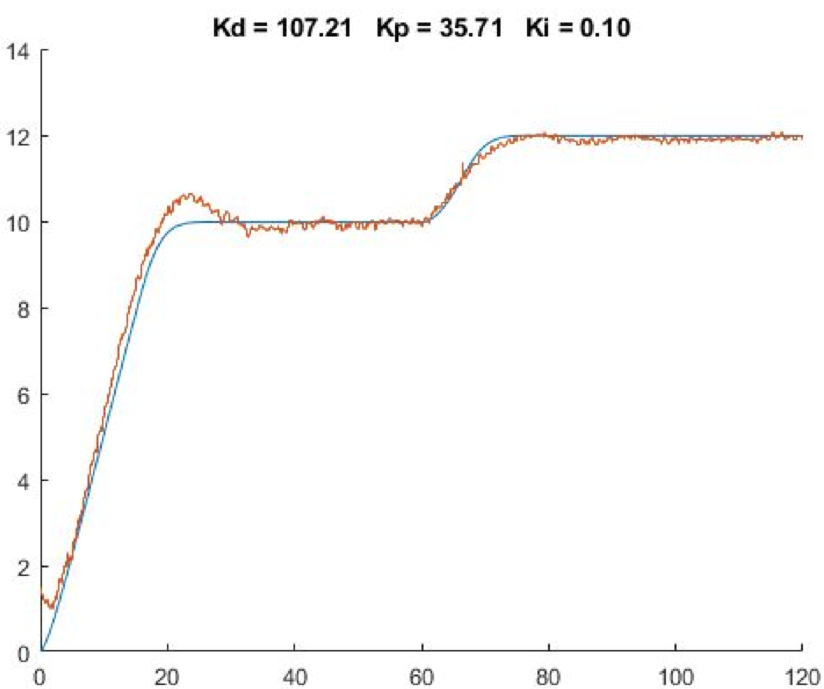

Figure 12. NonLinear Response with Optimized Parameters

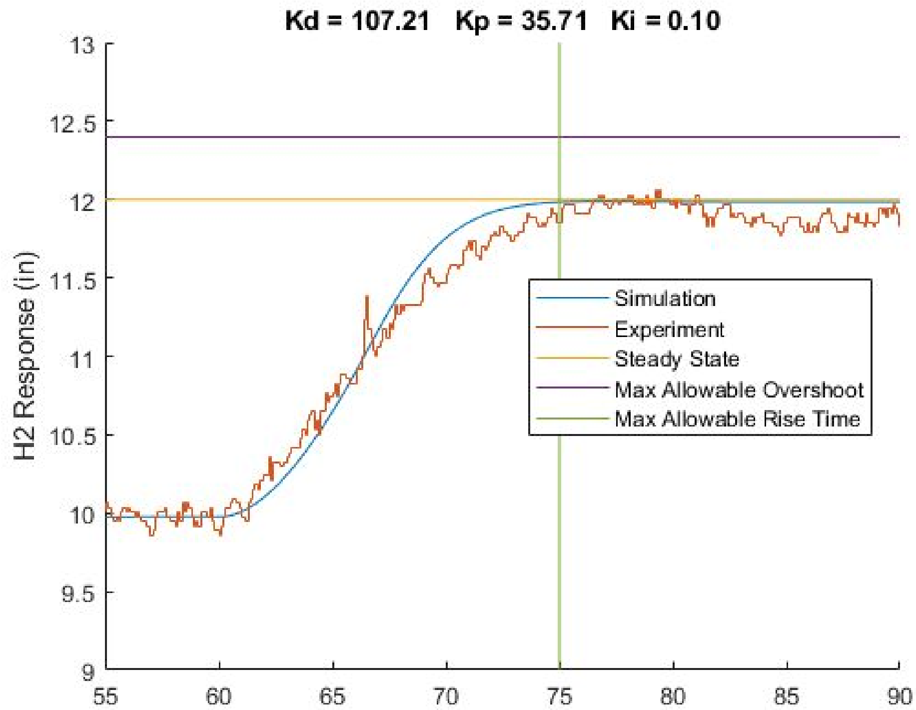

Figure 13. NonLinear Response with Optimized Parameters in Area of Interest

As shown, the models do very closely approximate the coupled tank system controlled by the PID controller. However, there are clear errors between the model and the experiment. This can be explained by errors in calibration constants or the limitations on the power output capable of the motor. Discrepancies due to the calibration constants is highly likely because it was observed that the measured height did not exactly match the height read by Simulink. Even after a second calibration, this was still an issue because the calibration was performed without flowing water. It is conjectured that the falling water increased the pressure observed by the transducer, which could explain why the observed height was usually within ~0.5cm below the Simulink recorded water heights. As shown above, it was possible to achieve the performance specifications given in this lab. However, it is not possible to achieve every combination of performance parameters due to the limit on the power and sensor sampling speed. Therefore, increasing certain gains will have no effect after a certain point as the power supply and motor cannot perform in a way that the controller has specified. Along the same lines, the saturation block does affect the PID controller as it limits the ability of the controller. The saturation block limits the voltage output of the controller to be within the capabilities of the power supply and motor. This is necessary to preserve the system, but in the same reasoning as before, can limit the capability of the controller to meet certain performance specifications.

VI. Conclusions

In conclusion, these three labs covered the calibration and simulation of a coupled tank system, the tuning of the PID controller to meet performance specifications, and the implementation of a PID controller to regulate the height of water in the tanks. Using the identified parameters and calibration, the controller gains were estimated to be as follows: K_p = 6.7135; K_i = 1.0078, and K_d = 67.2116. Adjusting the parameters to bring the linear model within the given specifications gave the following gains: K_p = 35.7135; K_i = 0.0700, and K_d = 107.2116. Finally, because the controller for the linear model does not meet the performance parameters, adjusting the gains to bring the nonlinear model within the given specifications gave the following gains: K_p = 35.7135; K_i = 0.1000, and K_d = 107.2116. After implementing the controllers with each of the above sets of gains, it was observed that the real world system very closely matches the simulated response, with the nonlinear model best estimating performance. Discrepancies between the model and simulation can be explained by the limitations of the motor and power supply as well as issues derived from calibrating the transducer with static rather than flowing water. Overall, these labs strengthened the skills of calibration and deriving models, while exemplifying control design procedure and the implementation of digital controllers.

VII. References

[1] Alladi, Vijay et. al. (2001). *MEEN 364 Lab Manual: Lab 5 -Quanser Coupled Tanks Modeling and Parameter Identification.* Texas A&M University. Web.

[2] Alladi, Vijay et. al. (2001). *MEEN 364 Lab Manual: Lab 8 - Design and Simulation of Controller for Quanser Coupled Tanks System.* Texas A&M University. Web.

[3] Alladi, Vijay et. al. (2001). *MEEN 364 Lab Manual: Lab 9 - Implementation of Controller for Coupled Tanks System.* Texas A&M University. Web.

VIII. Appendices

Appendix A. MatLab Code.

analysis.m

```
clc; clear;

%% Estimation

% Original Variable Estimates
run estimate_param.m

KpL = Kpm;
KdL = Kdm;
KiL = Kim;
sim Lab8Linear_est
Sim_est = simout;
Data_est = load('lab9_estimated.mat');

figure(1);
hold on
plot(Sim_est)
plot(Data_est.simout)
ttl = sprintf('Kd = %.2f   Kp = %.2f   Ki = %.2f',KdL,KpL,KiL);
title(ttl);

figure(2);
hold on
plot(Sim_est)
plot(Data_est.simout)
plot([0 600],[12 12],[0 600],[12+2*.2 12+2*.2],[315 315],[0 20]);
axis([290 450 9 14]);
ylabel('H2 Response (in)');
title(ttl);
legend('Simulation','Experiment','Steady State','Max Allowable Overshoot',...
   'Max Allowable Rise Time','Location','Best');

%% Linearly Optimized

% Linear Optimized Variables
KpL = 35.7135;
KdL = 107.2116;
KiL = 0.07;
sim Lab8Linear;
Sim_lin = simout;
Data_lin = load('lab9_linoptimized.mat');
```

```
figure(3);
hold on
plot(Sim_lin)
plot(Data_lin.simout)
ttl = sprintf('Kd = %.2f   Kp = %.2f   Ki = %.2f',KdL,KpL,KiL);
title(ttl);

figure(4);
hold on
plot(Sim_lin)
plot(Data_lin.simout)
plot([0 90],[12 12],[0 90],[12+2*.2 12+2*.2],[75 75],[0 20]);
axis([55 90 9 13]);
ylabel('H2 Response (in)');
title(ttl);
legend('Simulation','Experiment','Steady State','Max Allowable Overshoot',...
    'Max Allowable Rise Time','Location','Best');


%% Nonlinearly Optimized

%NonLinear Optimized Variables
KpL = 35.7135;
KdL = 107.2116;
KiL = 0.1;
sim Lab8NonLinear;
Sim_non = simout;
Data_non = load('lab9_nonoptimized.mat');

figure(5);
hold on
plot(Sim_non)
plot(Data_non.simout)
ttl = sprintf('Kd = %.2f   Kp = %.2f   Ki = %.2f',KdL,KpL,KiL);
title(ttl);

figure(6);
hold on
plot(Sim_non)
plot(Data_non.simout)
plot([0 90],[12 12],[0 90],[12+2*.2 12+2*.2],[75 75],[0 20]);
axis([55 90 9 13]);
ylabel('H2 Response (in)');
title(ttl);
```

```
legend('Simulation','Experiment','Steady State','Max Allowable Overshoot',...
    'Max Allowable Rise Time','Location','Best');
```

estimate_param.m

```
%Values found from Lab 5
C1 = 6.5865;
C2 = 6.8562;
k = 1.9756;
ks = 2.2493;
A = 15.9;

h20 = 10;
qi0 = C2*sqrt(h20);
h10 = (qi0/C1)^2;

Gv1 = C1/(2*sqrt(h10));
Gv2 = C2/(2*sqrt(h20));

alpha = (Gv1*k)/A^2;
beta = (Gv1+Gv2)/A;
gamma = (Gv1*Gv2)/A^2;

Overshoot = 0.20;
Rise_time = 15;

zeta = sqrt((log(Overshoot)^2)/(pi^2+log(Overshoot)^2));
wn = 1.8/Rise_time;

Kdm = (12*zeta*wn-beta)/alpha;
Kpm = 0.75*(wn^2 + 20*zeta^2*wn^2-gamma)/alpha;
Kim = 10*zeta*wn^3/alpha;
```

Lab8.m

```
clc; clear;

run estimate_param.m

% Original Variable Estimates
KpL = Kpm;
KdL = Kdm;
KiL = Kim;
sim Lab8NonLinear
```

```
% % Linear Optimized Variables
% KpL = 35.7135;
% KdL = 107.2116;
% KiL = 0.07;
% sim Lab8Linear

% %NonLinear Optimized Variables
% KpL = 35.7135;
% KdL = 107.2116;
% KiL = 0.1;
% sim Lab8NonLinear

ttl = sprintf('Response\nKd = %.2f\nKp = %.2f\nKi = %.2f',KdL,KpL,KiL);

% % plot just the data
% plot(simout)
% legend(ttl,'Location','Best');

% % plot SS and max overshoot lines
plot(simout,[0 600],[12 12],[0 600],[12+2*.2 12+2*.2],[315 315],[0 20]);
legend(ttl,'Steady State','Max Allowable Overshoot','Max Allowable Rise Time','Location','Best');

% Zoom to area of interest
axis([290 350 10 13])

% Fix Labels for Report
ylabel('H2 Response (in)');
title('');
```

Lab9.m

```
clc; clear;

% Original Variable Estimates
KpL = 6.7135;
KdL = 67.2116;
KiL = 1.0078;

% % Linear Optimized Variables
% KpL = 35.7135;
% KdL = 107.2116;
% KiL = 0.07;

%NonLinear Optimized Variables
```

```
% KpL = 35.7135;
% KdL = 107.2116;
% KiL = 0.1;

sim Lab9
plot(simout)
```

Appendix B. Simulink Code.

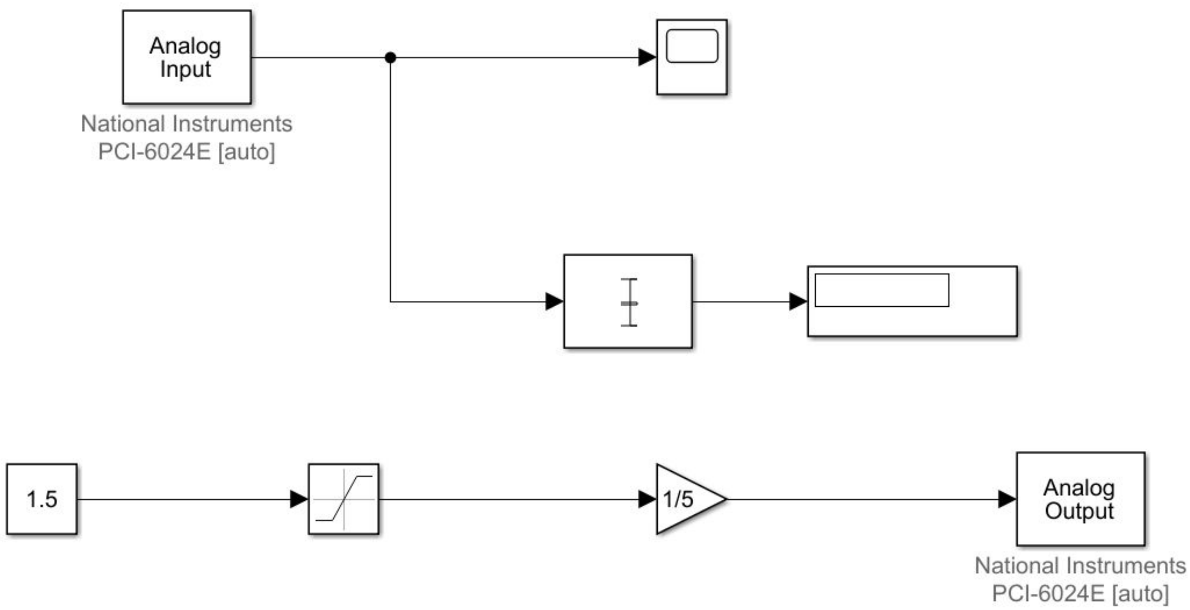

Figure 14. Lab 5 Simulink Code

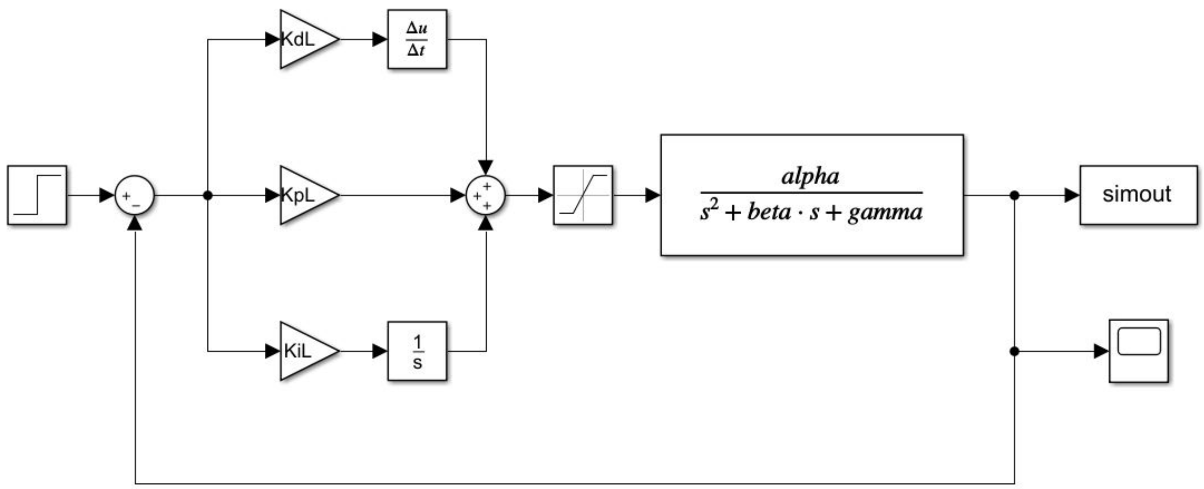

Figure 15. Lab 8 Linear Model Simulink Code

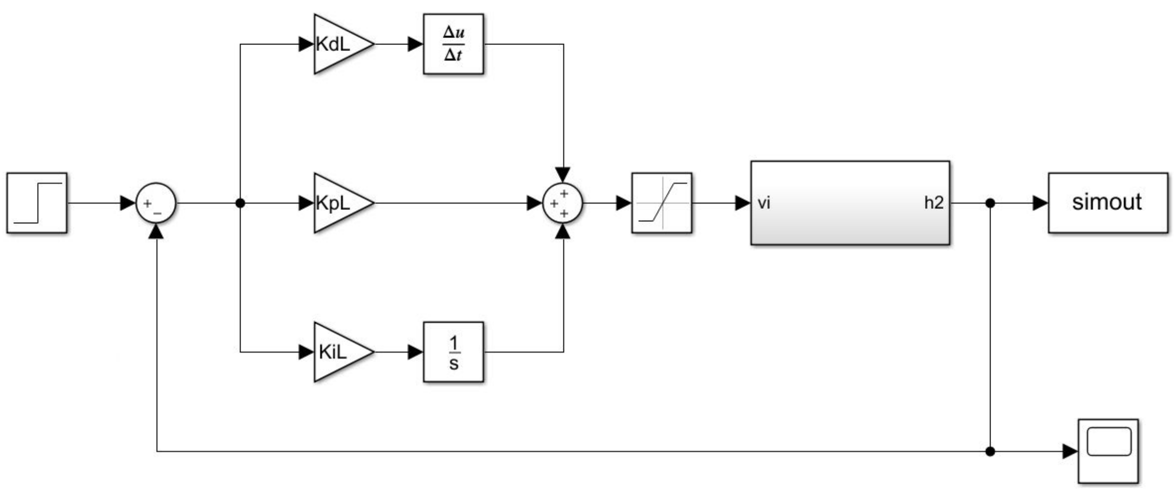

Figure 16. Lab 8 NonLinear Simulink Code

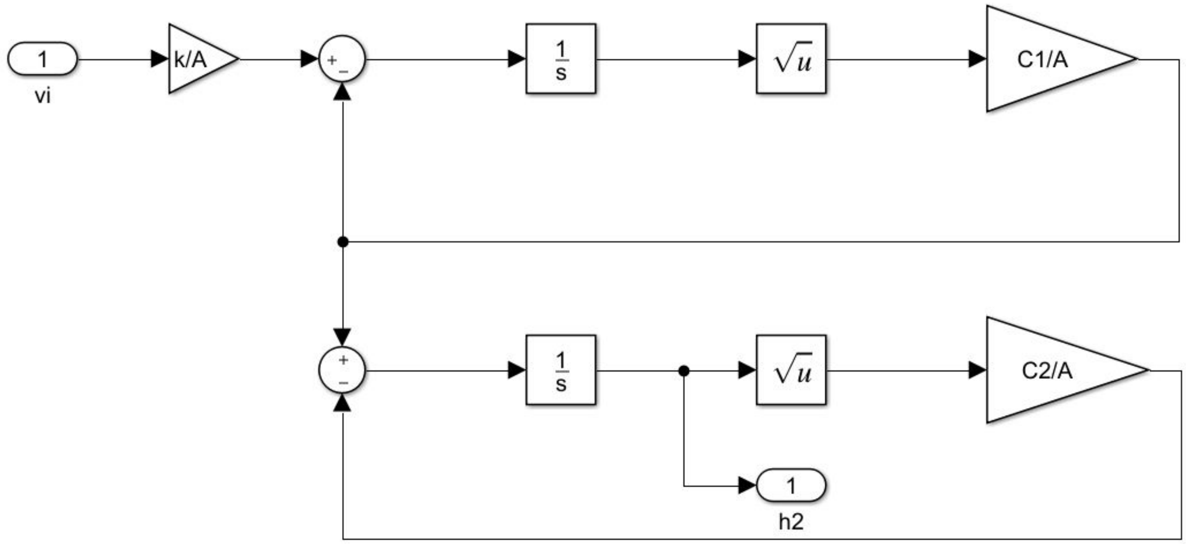

Figure 17. Lab 8 Nonlinear Model Subsystem

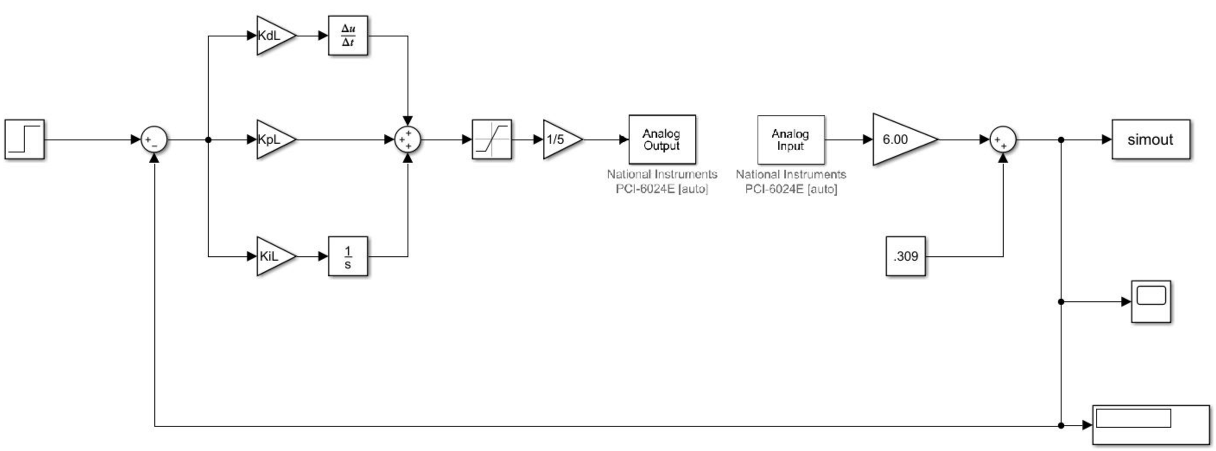

Figure 18. Lab 9 Simulink Controller

Appendix C. Calculations

Derivation of non-linear system for lab 8:

$q_i - c_1\sqrt{h_1} - A_1 * \frac{dh_1}{dt} = 0$

$c_1\sqrt{h_1} - c_2\sqrt{h_2} - A_2 * \frac{dh_2}{dt} = 0$

$q_i = k_m v_i$

$A_1 = A_2 = A$

$k_m v_i - c_1\sqrt{h_1} - A_1 * \frac{dh_1}{dt} = 0$

$$c_1\sqrt{h_1} - c_2\sqrt{h_2} - A_2 * \frac{dh_2}{dt} = 0$$

$$\frac{dh_1}{dt} = \frac{k_m}{A}v_i - \frac{c_1}{A}\sqrt{h_1}$$

$$\frac{dh_1}{dt} = \frac{c_1}{A}\sqrt{h_1} - \frac{c_2}{A}\sqrt{h_2}$$

Appendix D. Raw Data and Figures.

Table 3. Pump Calibration Raw Data

Voltage	Area of Base	Height 1	h1^1/2	Height 2	h2^1/2	Time to 20cm	Vol.Flow Rate
(V)	(cm^2)	(cm)	(cm)^1/2	(cm)	(cm)^1/2	(s)	(cm^3/s)
8	15.9	4.5	2.121320344	4	2	19.84	16.02822581
10	15.9	9	3	8.5	2.915475947	15.88	20.02518892
12	15.9	13	3.605551275	12.5	3.535533906	13.28	23.94578313
15	15.9	20.5	4.527692569	19.5	4.415880433	11.1	28.64864865
18	15.9	30	5.477225575	26.5	5.14781507	8.84	35.97285068