Modeling and Analysis of the Omron Adept Hornet 565 Delta Robot –MEEN 612 Final Project–

Mathias Bendixen
Mechanical Engineering
Texas A&M University
College Station, TX 77843
Email: bendixenm@tamu.edu

Larsen Dempsey
Mechanical Engineering
Texas A&M University
College Station, TX 77843
Email: lars527736@tamu.edu

Jacob Hartzer
Mechanical Engineering
Texas A&M University
College Station, TX 77843
Email: jmhartzer@tamu.edu

***Abstract*—This report summarizes the author's term project for MEEN 612: Mechanics of Robot Manipulators. This report concerns the kinematics and dynamics as well as simulation study of an Omron Adept Hornet 565 four degree-of-freedom delta robot. Using a loop-closure equation, the forward kinematics were derived. The inverse kinematics were derived making use of the tangent half-angle substitution. The Jacobian matrix was derived by differentiating the loop-closure equation and making use of the constraints on the upper arm. The mass matrix was derived by solving the inertia matrices for each component by relating rotational and transnational kinetic energy frames and using the Jacobian. Finally, MatLab code was written in conjunction to the creation of solid body models to find the limitations of the joint and operational space as well as simulate linearized trajectory planning.**

I. Introduction

The Omron Adept Hornet 565 Delta Robot is a popular parallel delta robot used for high-speed picking and packaging applications. With a nominal cycle time of 0.32 seconds, this robot is idea for many pick and place operations where speed and precision are required [1]. The Hornet 565 is shown in Figure 1. As shown, the Hornet is a delta robot with three parallel joints and a revolute joint at the base platform. This robot was chosen for the subject of this report due to the wide number of usage cases, as well as its popularity in industry.

Fig. 1. Omron Adept Hornet 565

II. Kinematics

A. Forward Kinematics [2]

The Omron 565 Delta robot manipulator consists of two parallel platforms connected by three linkages. These linkages are composed of one revolute arm and two reactionary parallelogram-forming links. The revolute joint is connected to the motor and is therefore forced to rotate in plane, however the parallelogram-forming links are connected by ball and socket joints, and are free to rotate. These linkages and the shaft that controls the end effector rotation will be modeled as solid cylindrical rods.

The kinematics of this robot can be studied by dividing the robot into three closed loop chains. A single loop chain is shown in Figure 2 where O is the origin of the manipulator, A_i is one of the origins of the revolute joints, B_i is the connection between the first and second arms, C_i is the connection of the base and the second arm, and P is the center of the end effector platform. Four coordinate systems are defined xyz and $x_iy_iz_i$ such that the origin of all four is identical to O, the xy and x_iy_i planes are identical and coincide with the plane of the fixed platform, and the angle between x and x_i is defined as ϕ_i. Axes z and z_i are consequently also identical.

Due to the ball and socket joints located between the first and second joints, the second joint is not constrained to the plane of the revolute joint like the first joint is. Therefore, angle θ_{1i} is defined as the angle between $\overrightarrow{A_iB_i}$ and $\overrightarrow{A_ix_i}$ and θ_{2i} is defined as the angle between $\overrightarrow{A_ix_i}$ and the projection of $\overrightarrow{B_iC_i}$ on the x_iz_i plane. Lastly, as shown in Figure 3, θ_{3i} is defined as the angle between y_i and $\overrightarrow{B_iC_i}$. Note that $\overrightarrow{B_iC_i}$ is typically not in plane with any of the coordinate systems.

In this system, the revolute joints control the values of θ_{1i}, and the operational space is in terms of point P. Therefore,

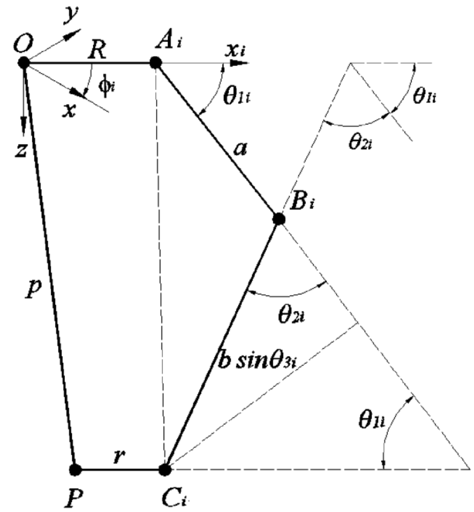

Fig. 2. Projection of Single Loop Chain onto Revolute Plane

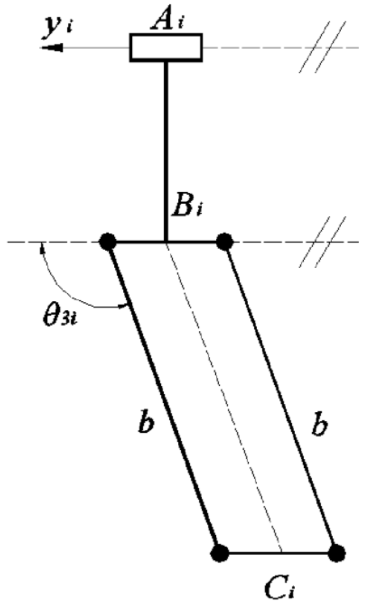

Fig. 3. Perpendicular View of Single Loop Chain

the obvious choice for closed loop kinematics involves the following

$$\vec{\boldsymbol{\theta}} = \begin{bmatrix} \theta_{11} \\ \theta_{12} \\ \theta_{13} \end{bmatrix}, \vec{\boldsymbol{p}} = \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix}, \tag{1}$$

To derive the kinematic equations, a generic closed loop $OA_iB_iC_iPO$ is used. Written out in vector notation this is

$$\overrightarrow{\boldsymbol{OP}} + \overrightarrow{\boldsymbol{PC}_i} = \overrightarrow{\boldsymbol{OA}_i} + \overrightarrow{\boldsymbol{A}_i\boldsymbol{B}_i} + \overrightarrow{\boldsymbol{B}_i\boldsymbol{C}_i} \tag{2}$$

This vector equation can then be broken into components in the $x_iy_iz_i$ frame. Written in matrix form, this equation is

$$\begin{bmatrix} p_x\cos\phi_i - p_y\sin\phi_i \\ p_x\sin\phi_i + p_y\cos\phi_i \\ p_z \end{bmatrix} = \begin{bmatrix} R \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} r \\ 0 \\ 0 \end{bmatrix} + a\begin{bmatrix} \cos\theta_{1i} \\ 0 \\ \sin\theta_{1i} \end{bmatrix} + b\begin{bmatrix} \sin\theta_{3i}\cos(\theta_{2i}+\theta_{1i}) \\ \cos\theta_{3i} \\ \sin\theta_{3i}\sin(\theta_{2i}+\theta_{1i}) \end{bmatrix} \tag{3}$$

In order to find the Jacobian, these equations are differentiated with respect to time, as outlined in the next section.

B. Inverse Kinematics [2]

In the case of the Delta robot, the inverse kinematic solution is rather straightforward. Using a method similar to loop closure, the problem at hand can be reduced to solving for the intersection of each of three circles with a sphere. To begin, the closed loop equations are of the form

$$E_i\cos\theta_{1i} + F_i\sin\theta_{1i} + G_i = 0 \quad i = 1,2,3 \tag{4}$$

Expanded into matrix form

$$a\begin{bmatrix} 2(y+\alpha) \\ -(\sqrt{3}(p_x+\beta)+p_y+\gamma) \\ -(\sqrt{3}(p_x-\beta)-p_y-\gamma) \end{bmatrix}\cdot\cos\theta_{1i} + \begin{bmatrix} 2p_za \\ 2p_za \\ 2p_za \end{bmatrix}\sin\theta_{1i} + \begin{bmatrix} r^2+\alpha^2+2p_y\alpha \\ r^2+\beta^2+\gamma^2+2(p_x\beta+p_yp_z) \\ r^2+\beta^2+\gamma^2+2(-p_x\beta+p_y\gamma) \end{bmatrix} = 0 \quad i = 1,2,3 \tag{5}$$

where

$$r^2 = p_x^2 + p_y^2 + p_z^2 + a^2 - b^2$$
$$\alpha = R - r$$
$$\beta = \frac{\sqrt{3}(r-R)}{2}$$
$$\gamma = \frac{r-R}{2}$$

This system of equations is solvable using the Tangent Half-Angle Substitution by defining the following

$$t_i = \tan\frac{\theta_{1i}}{2}$$
$$\cos\theta_{1i} = \frac{1-t_i^2}{1+t_i^2} \tag{6}$$
$$\sin\theta_{1i} = \frac{2t_i}{1+t_i^2}$$

Substitute Equation (6) into Equation (4) to produce

$$E_i\left(\frac{1-t_i^2}{1+t_i^2}\right) + F_i\left(\frac{2t_i}{1+t_i^2}\right) + G_i = 0 \tag{7}$$
$$E_i(1-t_i^2) + F_i(2t_i) + G_i(1+t_i^2) = 0$$

Using the quadratic formula, we can solve for t_i

$$t_i = \frac{-F_i \pm \sqrt{E_i^2+F_i^2-G_i^2}}{G_i - E_i} \tag{8}$$

Finally, solve for θ_{1i} using the original Tangent Half-Angle substitution

$$\theta_{1i} = 2\arctan t_i \tag{9}$$

Note that this methods provides two solutions to the inverse kinematic problem. In order to rectify this, constrain the system by always taking the minimum of these two solutions for each of the three joint variables. This enables a singular solution and is the correct configuration under the assumptions that the delta robot starts with the so called 'knees' of each joint bent away from the center and that the robot never reaches a singular position. Therefore, the 'knees' are always bent outward and the minimum of the two θ_{1i} values is correct.

III. Deriving the Jacobian [2]

The Jacobian will is derived by differentiating the closed loop kinematic equation and arranging the results in the following form

$$J_\theta \begin{bmatrix} \dot{\theta_{11}} \\ \dot{\theta_{12}} \\ \dot{\theta_{13}} \end{bmatrix} = J_p \begin{bmatrix} \dot{p_x} = v_x \\ \dot{p_y} = v_y \\ \dot{p_z} = v_z \end{bmatrix} \tag{10}$$

Where v_x, v_y, and v_z are the x, y, and z components of the velocity of point P, the center of the end effector platform. Before differentiating the forward kinematic equations, the following simplifications can be made.

$$\begin{aligned} \overrightarrow{\boldsymbol{OP}} &= \overrightarrow{\boldsymbol{p}} \\ \overrightarrow{\boldsymbol{PC}_i} &= \overrightarrow{\boldsymbol{r}} \\ \overrightarrow{\boldsymbol{OA}_i} &= \overrightarrow{\boldsymbol{R}} \\ \overrightarrow{\boldsymbol{A}_i\boldsymbol{B}_i} &= \overrightarrow{\boldsymbol{a}_i} \\ \overrightarrow{\boldsymbol{B}_i\boldsymbol{C}_i} &= \overrightarrow{\boldsymbol{b}_i} \end{aligned}$$

With these simplifications, Equation (2) can be rewritten as

$$(\overrightarrow{\boldsymbol{p}} + \overrightarrow{\boldsymbol{r}}) = \overrightarrow{\boldsymbol{R}} + \overrightarrow{\boldsymbol{a}_i} + \overrightarrow{\boldsymbol{b}_i} \tag{11}$$

Differentiating this new equation with respect to time produces the following

$$\dot{\overrightarrow{\boldsymbol{p}}} = \overrightarrow{\boldsymbol{v}} = \dot{\overrightarrow{\boldsymbol{a}}}_i + \dot{\overrightarrow{\boldsymbol{b}}}_i \tag{12}$$

Note that the $\overrightarrow{r}$ term goes to zero because it is a relationship between the center of the end effector platform and the connection points and is therefore constant. Using the rigidity of the end effector platform to show velocity at every point is constant, the following equation is found

$$\overrightarrow{\boldsymbol{v}} = \overrightarrow{\boldsymbol{\omega}_{a_i}} \times \overrightarrow{\boldsymbol{a}_i} + \overrightarrow{\boldsymbol{\omega}_{b_i}} \times \overrightarrow{\boldsymbol{b}_i} \tag{13}$$

This equation of linear velocities can be converted to an equation of angular velocities using the identity $v = r \times \omega$. However, when this equation is expanded to component form, it is apparent there is a dependence on $\dot{\theta}_{2i}$ and $\dot{\theta}_{3i}$. These variables prove rather difficult to find. However, this dependence can be removed by taking the dot product of Section III with the previously defined unit vector $\hat{\boldsymbol{b}}_i$.

$$\hat{\boldsymbol{b}}_i \cdot [\overrightarrow{\boldsymbol{v}} = \overrightarrow{\boldsymbol{\omega}_{a_i}} \times \overrightarrow{\boldsymbol{a}_i} + \overrightarrow{\boldsymbol{\omega}_{b_i}} \times \overrightarrow{\boldsymbol{b}_i}]$$

$$\hat{\boldsymbol{b}}_i \cdot \overrightarrow{\boldsymbol{v}} = \hat{\boldsymbol{b}}_i \cdot \overrightarrow{\boldsymbol{\omega}_{a_i}} \times \overrightarrow{\boldsymbol{a}_i}$$

Breaking this vector equation into components produces the following

$$\begin{aligned} \hat{\boldsymbol{b}_i} \cdot \overrightarrow{\boldsymbol{v}} =& [\cos(\theta_{1i} + \theta_{2i}) \sin\theta_{3i}][v_x \cos\phi_i - v_y \sin\phi_i] \\ &+ \cos\theta_{3i}[v_x \sin\phi_i + v_y \cos\phi_i] \\ &+ [\sin(\theta_{1i} + \theta_{2i}) \sin\theta_{3i}]v_z \end{aligned} \tag{14}$$

Therefore

$$\hat{\boldsymbol{b}_i} \cdot \overrightarrow{\boldsymbol{v}} = J_{ix}v_x + J_{iy}v_y + J_{iz}v_z \tag{15}$$

where

$$\begin{aligned} J_{ix} &= \cos(\theta_{1i} + \theta_{2i}) \sin\theta_{3i} \cos\phi_i + \cos\theta_{3i} \sin\phi_i \\ J_{iy} &= -\cos(\theta_{1i} + \theta_{2i}) \sin\theta_{3i} \sin\phi_i + \cos\theta_{3i} \sin\phi_i \\ J_{iz} &= \sin(\theta_{1i} + \theta_{2i}) \sin\theta_{3i} \end{aligned} \tag{16}$$

By definition of the axis relative to the revolute joints, it is known that joint a is constrained to movement in the x_iz_i plate. Thus

$$\overrightarrow{\boldsymbol{\omega}_{a_i}} = \begin{bmatrix} 0 \\ -\dot{\theta}_{1i} \\ 0 \end{bmatrix} \tag{17}$$

Therefore, $\overrightarrow{v}$ can be redefined as

$$\overrightarrow{\boldsymbol{\omega}}_{a_i} \times \overrightarrow{\boldsymbol{a}}_i = \begin{vmatrix} \hat{\boldsymbol{\imath}} & \hat{\boldsymbol{\jmath}} & \hat{\boldsymbol{k}} \\ 0 & -\dot{\theta}_{1i} & 0 \\ a_{1i} & a_{2i} & a_{3i} \end{vmatrix} = -a_{3i}\dot{\theta}_{1i}\hat{\boldsymbol{\imath}} + a_{1i}\dot{\theta}_{1i}\hat{\boldsymbol{k}}$$

Reintroducing the full left-hand side of Equation (14) gives

$$\hat{\boldsymbol{b}_i} \cdot (\overrightarrow{\boldsymbol{\omega}_{a_i}} \times \overrightarrow{\boldsymbol{a}_i}) = -a \sin\theta_{2i} \sin\theta_{3i}\dot{\theta}_{1i} \tag{18}$$

Expanding this vector equation into its components produces the following

$$\begin{aligned} J_{1x}v_x + J_{1y}v_y + J_{1z}v_z &= -a \sin\theta_{21} \sin\theta_{31}\dot{\theta_{11}} \\ J_{2x}v_x + J_{2y}v_y + J_{2z}v_z &= -a \sin\theta_{22} \sin\theta_{32}\dot{\theta_{12}} \\ J_{3x}v_x + J_{3y}v_y + J_{3z}v_z &= -a \sin\theta_{23} \sin\theta_{33}\dot{\theta_{13}} \end{aligned} \tag{19}$$

These equations can be rewritten in the originally desired form

$$\boldsymbol{J_p}\overrightarrow{\boldsymbol{v}} = \boldsymbol{J_\theta}\dot{\overrightarrow{\boldsymbol{\theta}}} \tag{20}$$

where

$$\boldsymbol{J_p} = \begin{bmatrix} J_{1x} & J_{1y} & J_{1z} \\ J_{2x} & J_{2y} & J_{2z} \\ J_{3x} & J_{3y} & J_{3z} \end{bmatrix} \tag{21}$$

and

$$\begin{aligned} \boldsymbol{J_\theta} &= a \\ &\times \begin{bmatrix} \sin\theta_{21}\sin\theta_{31} & 0 & 0 \\ 0 & \sin\theta_{22}\sin\theta_{32} & 0 \\ 0 & 0 & \sin\theta_{23}\sin\theta_{33} \end{bmatrix} \end{aligned} \tag{22}$$

Therefore, the Jacobian matrix can be written as

$$\begin{aligned}\boldsymbol{J} =&\boldsymbol{J_p}^{-1}\boldsymbol{J_\theta}\\ =&\begin{bmatrix} J_{1x} & J_{1y} & J_{1z}\\ J_{2x} & J_{2y} & J_{2z}\\ J_{3x} & J_{3y} & J_{3z}\end{bmatrix}^{-1}\cdot a\\ &\times\begin{bmatrix}\sin\theta_{21}\sin\theta_{31} & 0 & 0\\ 0 & \sin\theta_{22}\sin\theta_{32} & 0\\ 0 & 0 & \sin\theta_{23}\sin\theta_{33}\end{bmatrix}\end{aligned} \tag{23}$$

IV. Dynamics

A. Mass Matrix

In order to find the mass matrix of the delta robot, it is first easier to establish the inertia matrix. This matrix is defined as the following sum of component inertias

$$\boldsymbol{H} = \boldsymbol{H_m} + \boldsymbol{H_{A1}} + \boldsymbol{H_{A2}} + \boldsymbol{H_{S1}} + \boldsymbol{H_{S2}} \tag{24}$$

where H_m is the inertia due to the base platform, H_{A1} is the inertia due to the first arm's resistance to rotation, H_{A2} is the inertia due to the second arm's resistance to rotational and transnational motion, H_{S1} is the inertia due to the upper shaft's resistance to rotation, and H_{S2} is the inertia due to the second shaft's resistance to rotation and translation. The simplest component inertia matrix is of the first links. This matrix is the rotational resistance to acceleration. Due to the simple geometry where each of these joints is constrained to the plane of rotation, each element is simply of the form $I = \frac{1}{3}mL^2$

$$\boldsymbol{H_{A1}} = \begin{bmatrix}\frac{1}{3}m_{A1}a^2 & 0 & 0\\ 0 & \frac{1}{3}m_{A1}a^2 & 0\\ 0 & 0 & \frac{1}{3}m_{A1}a^2\end{bmatrix} \tag{25}$$

In order to find the inertia matrix component due to the base plate, the definition of the Jacobian is used.

$$\dot{\boldsymbol{p}} = \boldsymbol{J}\dot{\boldsymbol{q}} \tag{26}$$

Because kinetic energy is the same regardless of reference frame

$$\begin{aligned}\frac{1}{2}\dot{\boldsymbol{q}}^T\boldsymbol{H_m}\dot{\boldsymbol{q}} &= \frac{1}{2}m_m\dot{\boldsymbol{p}}^T\dot{\boldsymbol{p}}\\ &= \frac{1}{2}m_m\dot{\boldsymbol{q}}^T\boldsymbol{J}^T\boldsymbol{J}\dot{\boldsymbol{q}}\\ &= \frac{1}{2}\dot{\boldsymbol{q}}^T(m_m\boldsymbol{J}^T\boldsymbol{J})\dot{\boldsymbol{q}}\end{aligned} \tag{27}$$

Therefore

$$\boldsymbol{H_m} = \dot{m}_m\boldsymbol{J}^T\boldsymbol{J} \tag{28}$$

In order to determine the values of the matrix $\boldsymbol{H_{A2}}$ a similar approach will be used as in determining the $\boldsymbol{H_m}$ matrix in Equation (27) by using the total kinetic energy

$$\frac{1}{2}\dot{\boldsymbol{q}}^T\boldsymbol{H_{A2}}\dot{\boldsymbol{q}} = \boldsymbol{T}_{H2} \tag{29}$$

This can be rewritten in matrix form as follows

$$\frac{1}{2}\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\\ \dot{q}_3\end{bmatrix}^T\begin{bmatrix}H_{A2_1} & 0 & 0\\ 0 & H_{A2_2} & 0\\ 0 & 0 & H_{A2_3}\end{bmatrix}\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\\ \dot{q}_3\end{bmatrix} = \begin{bmatrix}T_{A2_1}\\ T_{A2_2}\\ T_{A2_3}\end{bmatrix} \tag{30}$$

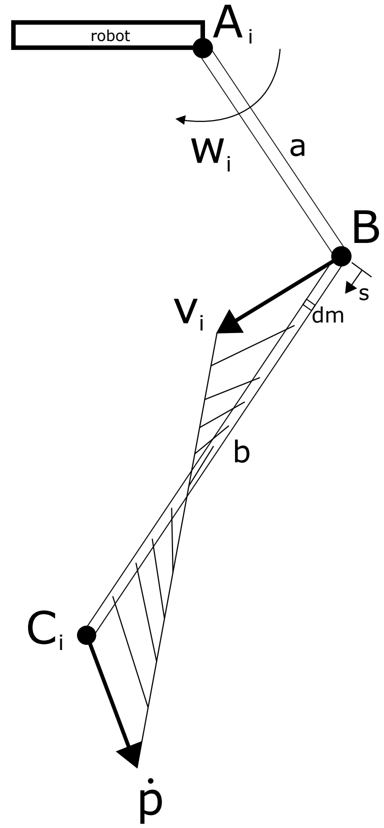

Fig. 4. Kinetic Energy Integration Across Second Link

This equation can be simplified as follows

$$\frac{1}{2}\begin{bmatrix}(\dot{q}_1)^2H_{A2_1}\\ (\dot{q}_2)^2H_{A2_2}\\ (\dot{q}_3)^2H_{A2_3}\end{bmatrix} = \begin{bmatrix}T_{A2_1}\\ T_{A2_2}\\ T_{A2_3}\end{bmatrix} \tag{31}$$

In order to solve for the kinetic energy of each of the second links, consider Figure 4. Because the lower ball joint of the second arm is connected to the base platform, that point should have an velocity identical to the base platform, $\dot{\boldsymbol{p}}$. Additionally, the velocity of the higher ball joint can be found knowing the angular velocity and length of the joint.

$$v_i = \omega \times \overrightarrow{A_iB_i} \tag{32}$$

Therefore, the component velocities of a given point B_i are

$$\begin{aligned}&\omega_i\times\begin{bmatrix}a\sin\theta_{1i}\cos\phi_i\\ a\sin\theta_{1i}\sin\phi_i\\ a\cos\theta_{1i}\end{bmatrix}\\ &=\begin{vmatrix}\hat{\boldsymbol{i}} & \hat{\boldsymbol{j}} & \hat{\boldsymbol{k}}\\ \dot{\theta}_{1i}\sin\phi_i & \dot{\theta}_{1i}\cos\phi_i & 0\\ a\sin\theta_{1i}\cos\phi_i & a\sin\theta_{1i}\sin\phi_i & a\cos\theta_{1i}\end{vmatrix}\end{aligned} \tag{33}$$

$$\boldsymbol{V_i} = a\dot{\theta}_{1i}\begin{bmatrix}\cos\phi_i\cos\theta_{1i}\\ -\cos\phi_i\sin\theta_{1i}\cos\phi_i\\ \sin\theta_{1i}(\sin^2\phi_i - \cos^2\phi_i)\end{bmatrix} \tag{34}$$

Knowing the initial and final velocities along the rod, it is possible to parameterize the velocity as a function of an integration factor s as shown

$$\begin{bmatrix} V_x(s) \\ V_y(s) \\ V_z(s) \end{bmatrix} = \begin{bmatrix} v_{i_x} \\ v_{i_y} \\ v_{i_z} \end{bmatrix} \left(\frac{b-s}{b}\right) + \begin{bmatrix} \dot{p}_x \\ \dot{p}_y \\ \dot{p}_z \end{bmatrix} \left(\frac{s}{b}\right) \tag{35}$$

Using the kinetic energy equation $T = \frac{1}{2}mv^2$, the following integral is formulated.

$$\int_{A2_i} \frac{v^2}{2} dm \tag{36}$$

Assuming the linear mass density of the second arm is constant, the following substitution may be made

$$dm = \frac{m}{b} ds \tag{37}$$

Expanding the velocity vector into it's components gives

$$\frac{m}{2b} \int_0^b V_x^2 + V_y^2 + V_z^2 ds \tag{38}$$

Equation (39) shows the substitution of Equation (35) into the components of V_x^2. This procedure can be repeated for similarly find V_y^2 and V_z^2.

$$\begin{aligned} V_x^2 &= \left(V_{i_x}\left(\frac{b-s}{b}\right) + \dot{p}_x\left(\frac{s}{b}\right)\right)^2 \\ &= V_{i_x}^2\left(\frac{b-s}{b}\right)^2 + \dot{p}_x V_{i_x}\left(\frac{bs-s^2}{b^2}\right) + \dot{p}_x^2\left(\frac{s}{b}\right) \end{aligned} \tag{39}$$

Substituting Equation (39) into Equation (38) produces the following intergral.

$$\begin{aligned} T_i = \frac{m}{2b^2} \int_0^b & [V_{i_x}^2 + V_{i_y}^2 + V_{i_z}^2]\left(\frac{b-s}{b}\right) \\ & + [\dot{p}_x V_{i_x} + \dot{p}_y V_{i_y} + \dot{p}_z V_{i_z}]\left(\frac{bs-s^2}{b^2}\right) \\ & + [\dot{p}_x^2 + \dot{p}_y^2 + \dot{p}_z^2]\left(\frac{s}{b}\right)^2 ds \end{aligned} \tag{40}$$

Solving the integral gives the kinetic energy of each second joint.

$$\begin{aligned} T_{A2_i} = \frac{m}{12b} \cdot & [2(V_{i_x}^2 + V_{i_y}^2 + V_{i_z}^2) \\ & + (\dot{p}_x V_{i_x} + \dot{p}_y V_{i_y} + \dot{p}_z V_{i_z}) \\ & + 2(\dot{p}_x^2 + \dot{p}_y^2 + \dot{p}_z^2)] \end{aligned} \tag{41}$$

With these kinetic energies computed, it is possible to substitute into Equation (31) in order to find the following

$$\begin{bmatrix} H_{A2_1} \\ H_{A2_2} \\ H_{A2_3} \end{bmatrix} = 2 \begin{bmatrix} T_{A2_1}/(\dot{q}_1)^2 \\ T_{A2_2}/(\dot{q}_2)^2 \\ T_{A2_3}/(\dot{q}_3)^2 \end{bmatrix} \tag{42}$$

These values can be used to find

$$\boldsymbol{H_{A2}} = \begin{bmatrix} H_{A2_1} & 0 & 0 \\ 0 & H_{A2_2} & 0 \\ 0 & 0 & H_{A2_3} \end{bmatrix} \tag{43}$$

In order to solve the inertia of the upper shaft, it is necessary to find its rotation, ω_s. This can be derived from the following generic equation.

$$\omega = \frac{v \times \boldsymbol{r}}{|\boldsymbol{r}|^2} \tag{44}$$

Using the Jacobian, it is possible to substitute for $\boldsymbol{v}$.

$$\omega_S = \frac{\boldsymbol{J(q)\dot{q}} \times \boldsymbol{r}}{|\boldsymbol{r}|^2} \tag{45}$$

Using the same method as previously shown,

$$\frac{1}{2}\dot{\boldsymbol{q}}^T \boldsymbol{H_{S1}} \dot{\boldsymbol{q}} = \frac{1}{2}\boldsymbol{\omega_S^T I_{S1} \omega_S} \tag{46}$$

In matrix form

$$\begin{aligned} \frac{1}{2}\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix}^T \begin{bmatrix} H_{S1_1} & 0 & 0 \\ 0 & H_{S1_2} & 0 \\ 0 & 0 & H_{S1_3} \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix}^T \\ = \frac{1}{2}\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}^T \begin{bmatrix} I_{S1} & 0 & 0 \\ 0 & I_{S1} & 0 \\ 0 & 0 & I_{S1} \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} \end{aligned} \tag{47}$$

where

$$I_{S1} = \frac{1}{3} m_{S1} L_{S1}^2 \tag{48}$$

Therefore

$$\begin{aligned} H_{S1_1} &= I_{S1}\frac{\omega_1^2}{\dot{q_1}^2} \\ H_{S1_2} &= I_{S1}\frac{\omega_2^2}{\dot{q_2}^2} \\ H_{S1_3} &= I_{S1}\frac{\omega_3^2}{\dot{q_3}^2} \end{aligned} \tag{49}$$

Once these values have been found, they can be combined to calculate H_{R1}.

$$\boldsymbol{H_{S1}} = \begin{bmatrix} H_{S1_1} & 0 & 0 \\ 0 & H_{S1_2} & 0 \\ 0 & 0 & H_{S1_3} \end{bmatrix} \tag{50}$$

The final inertia matrix, H_{R2} can be found using a method similar to the second arm. As shown in Figure 5, the end of the lower shaft is attached to the end effector base, and therefore has an identical velocity. The top of the shaft points towards a point connected to the fixed plate, which has zero velocity. Therefore, there is a linear relationship between distance along the shaft and velocity. An integration variable s is defined starting from the base of the shaft, and the length of the shaft will be integrated with respect to mass.

$$\frac{1}{2}\dot{\boldsymbol{q}}^T \boldsymbol{H_{S2}} \dot{\boldsymbol{q}} = \boldsymbol{T_{S2}} \tag{51}$$

To find the kinetic energy of the lower shaft, a similar integral is computed

$$\int_{R2} \frac{v^2}{2} dm \tag{52}$$

Where, once again, the following substitution is made

$$dm = \frac{m}{b} ds \tag{37}$$

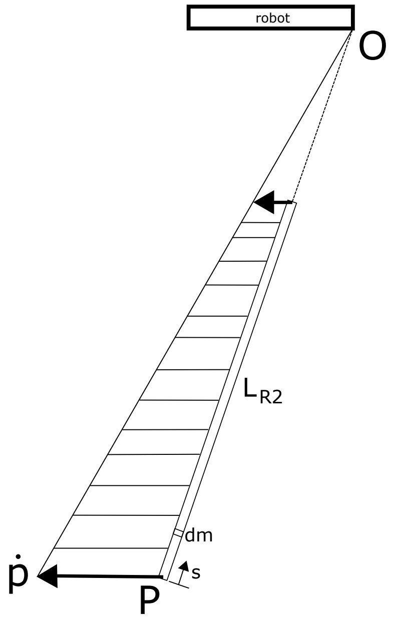

Fig. 5. Kinetic Energy Integration Across Lower Shaft

This produces the following integral

$$\boldsymbol{T_{S2}} = \frac{m}{2L}\int_0^L \left(\dot{\boldsymbol{p}}\cdot\left(\frac{|\boldsymbol{p}|-s}{|\boldsymbol{p}|}\right)\right)^2 ds \tag{53}$$

The values of $|V|^2$ and $|\boldsymbol{p}|$ are constant and therefore can be removed from the integral.

$$\frac{m\cdot|\boldsymbol{v}|^2}{L\cdot|\boldsymbol{p}|}\int_0^L (|\boldsymbol{p}|-s)\,ds \tag{54}$$

Solving the integral gives

$$\boldsymbol{T_{S2}} = \frac{m\cdot|\boldsymbol{v}|^2}{L\cdot|\boldsymbol{p}|}\left(|\boldsymbol{p}|L - \frac{1}{2}L^2\right) \tag{55}$$

With the kinetic energy computed, it is possible to substitute into Equation (51) in order to find the following

$$\begin{bmatrix} H_{S2_1} \\ H_{S2_2} \\ H_{S2_3} \end{bmatrix} = 2\begin{bmatrix} T_{S2}/(\dot{q}_1)^2 \\ T_{S2}/(\dot{q}_2)^2 \\ T_{S2}/(\dot{q}_3)^2 \end{bmatrix} \tag{56}$$

These values can be used to find

$$\boldsymbol{H_{S2}} = \begin{bmatrix} H_{S2_1} & 0 & 0 \\ 0 & H_{S2_2} & 0 \\ 0 & 0 & H_{S2_3} \end{bmatrix} \tag{57}$$

Finally, these component inertia matrices can be used to find the inertia matrix for the delta robot.

$$\boldsymbol{H} = \boldsymbol{H_m} + \boldsymbol{H_{A1}} + \boldsymbol{H_{A2}} + \boldsymbol{H_{S1}} + \boldsymbol{H_{S2}} \tag{24}$$

TABLE I
PARAMETERS FROM 3D MODELING

Parameter	Value and Units
m_{A1}	$2.030\ kg$
m_{A2}	$530.7\ g$
m_{S1}	$351.6\ g$
m_{S1}	$212.6\ g$
m_m	$1.436\ kg$
R	$227.3\ mm$
r	$75.0\ mm$
a	$315.0\ mm$
b	$825.0\ mm$
I_{A1}	$0.03747\ kg\cdot m^2$

Again, because kinetic energy is the same regardless of reference frame

$$\begin{aligned} \frac{1}{2}\dot{\boldsymbol{q}}^T\boldsymbol{H}(\boldsymbol{q})\dot{\boldsymbol{q}} &= \frac{1}{2}\dot{\boldsymbol{p}}^T\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{p}} \\ &= \frac{1}{2}(\boldsymbol{J}(\boldsymbol{q})\dot{\boldsymbol{q}})^T\boldsymbol{M}(\boldsymbol{q})(\boldsymbol{J}(\boldsymbol{q})\dot{\boldsymbol{q}}) \\ &= \frac{1}{2}\dot{\boldsymbol{q}}^T\boldsymbol{J}(\boldsymbol{q})^T\boldsymbol{M}(\boldsymbol{q})\boldsymbol{J}(\boldsymbol{q})\dot{\boldsymbol{q}} \\ \boldsymbol{H}(\boldsymbol{q}) &= \boldsymbol{J}(\boldsymbol{q})^T\boldsymbol{M}(\boldsymbol{q})\boldsymbol{J}(\boldsymbol{q}) \end{aligned} \tag{58}$$

Therefore

$$\boldsymbol{M}(\boldsymbol{q}) = \boldsymbol{J}(\boldsymbol{q})^{-T}\boldsymbol{H}(\boldsymbol{q})\boldsymbol{J}(\boldsymbol{q})^{-1} \tag{59}$$

B. Dynamic Equation

The final form of the dynamic equation will take the form

$$\boldsymbol{F} = \boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{x}} + \boldsymbol{N}(\boldsymbol{x},\dot{\boldsymbol{x}}) + \boldsymbol{G}(\boldsymbol{x}) \tag{60}$$

where x is the end effector position, $\boldsymbol{F}$ is the end effector force vector, $\boldsymbol{M}$ is the previously found mass matrix, $\boldsymbol{N}$ is the Centrifugal and Coriolis force matrix, and $\boldsymbol{G}$ is the gravitational force matrix. Determining $\boldsymbol{N}$ and $\boldsymbol{G}$ is beyond the scope of this report.

C. Results from Modeling

Using a three-dimensional solid model, it was possible to find a more accurate representation of the previously defined model. The found parameters are outlined in Table I and can be used to calculate the Jacobian and other parameters.

V. SIMULATION

A. Operational Space and Joint Space

Using MatLab, a simulation was created to model the singular positions. This was computed by finding end effector positions that made the Jacobian determinant ill-conditioned or zero. The results from this MatLab simulation are shown in Figure 6. These results were used alongside the solid modeling done in SolidWorks to find a generalized operational space shape shown in Figure 7. Additional, by noting the joint angles at the singular positions as well as possible collision positions, it was possible to find the generic limits of the Joint Space: $(-59° : 90°)$.

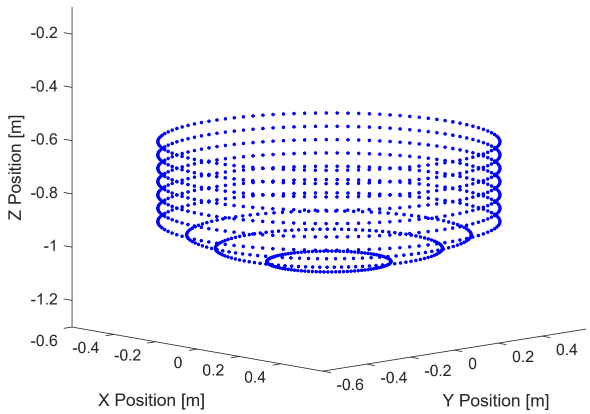

Fig. 6. MatLab Calculated Operational Space

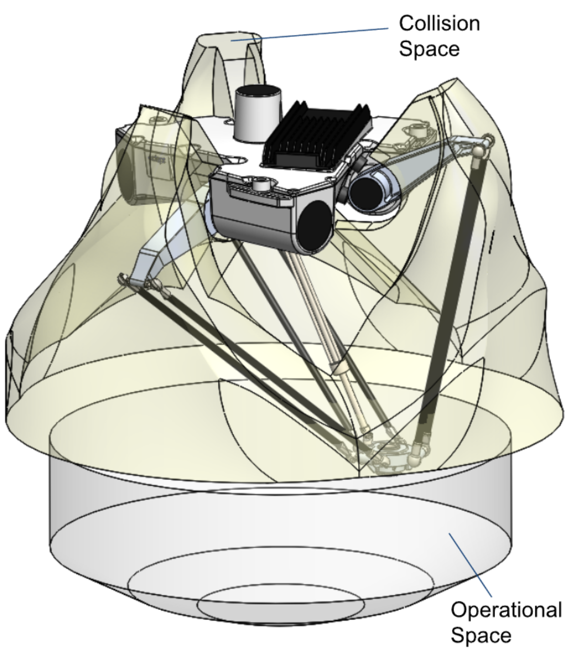

Fig. 7. Solid Body Model showing Operational Space and Collision Space

B. Linearized Trajectory Planning

Additionally, a MatLab simulation of linearized path planning was created. This simulation takes advantage of the inverse kinematic equations solved previously. This simulation takes a desired path, linearizes it to the desired precision, and then animates a model of the delta robot moving through the path. An example still from the output is shown in Figures 8 and 9

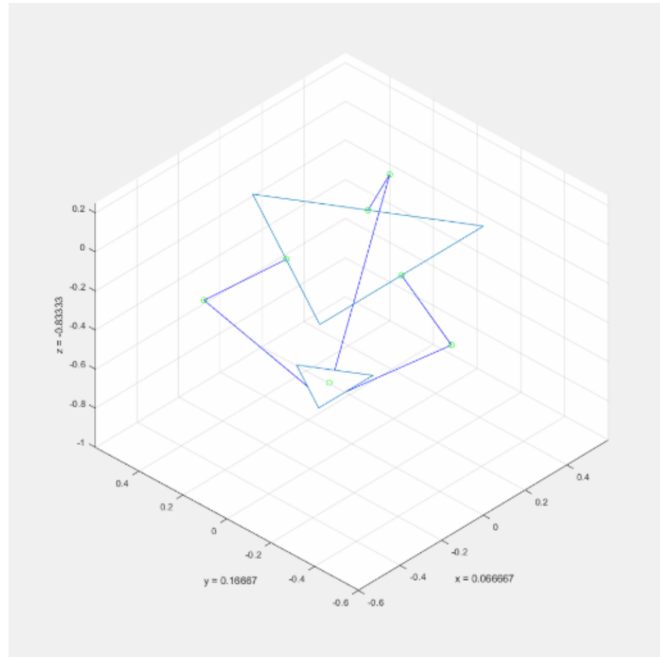

Fig. 8. Isometric View of Path Planning Simulation with Dimensions Shown in meters

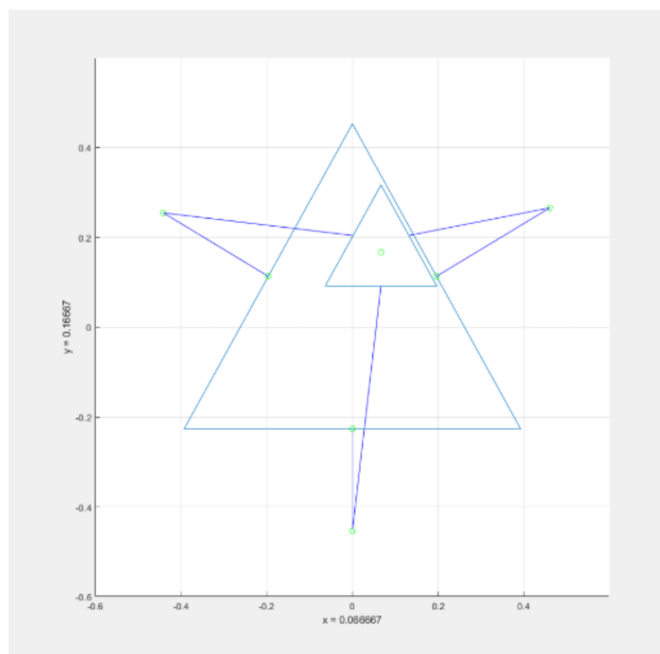

Fig. 9. Top-Down View of Path Planning Simulation with Dimensions Shown in meters

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